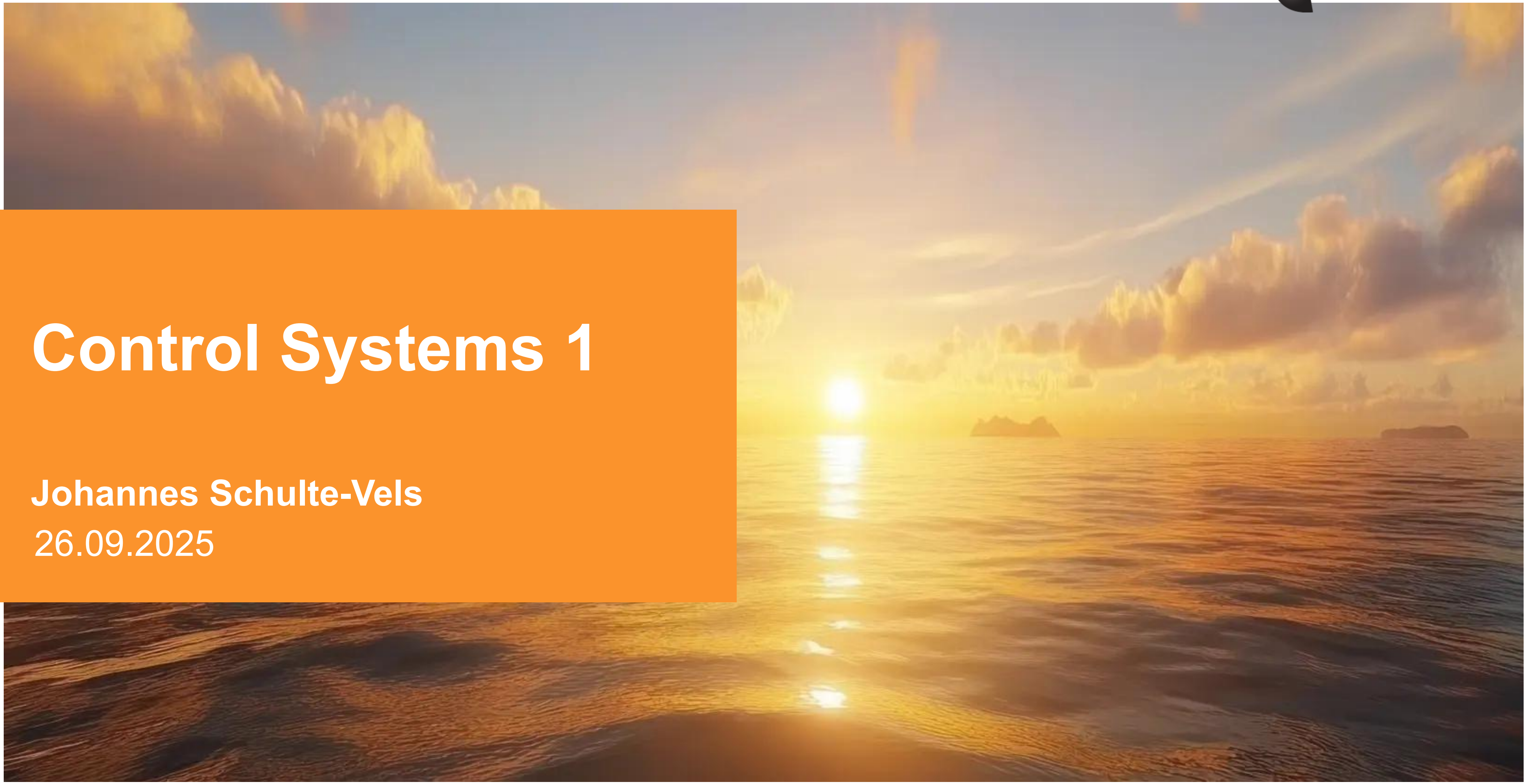
ETH zürich
Institute for Dynamic Systems and Control



Control Systems 1

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26.09.2025

Welcome!

Polybox

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Today

- Recap Session 1
- Theory Recap
  - Modeling
  - Block Diagrams
- Q&A Session / Done

Recap Session 1

What controll architecture is best suited for keeping a plane on a certain altitude?

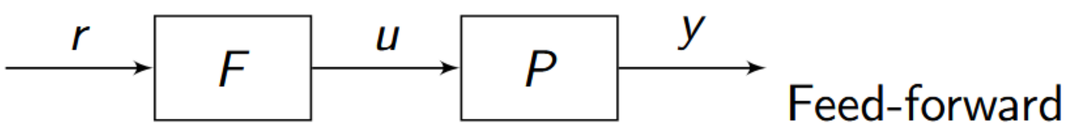
A) Open-Loop

B) Half-Open-Loop

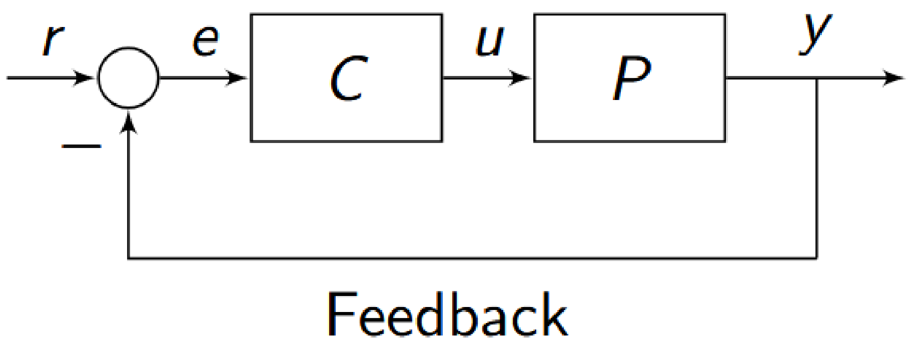
C) Closed-Loop

D) Super-Loop

System Comparison



- No feedback ➞ Input doesn't depend on output
- Simple but unprecise



- Feedback! ➞ Input depends on output
- More complex
- Can become unstable (we will later look at what that means)

What naming convention do we use?

A) **Input**: i(t); **Output**: o(t); **System**: σ

B) **Input:** u(t); **Output:** y(t); **System**: Σ

C) **Input:** x(t); **Output:** y(t); **System:** S

D) **Input:** u(t); **Output**: v(t); **System**: ζ

What are the main objectives of a controler?

A) Performance, Stability, Linearity

B) Linearity, Predictability, Affordability

C) Performance, Robustness, Stability

D) Data Analysis, Efficiency, Tracking

What type of system is this?

A) SISO

B) MIMO

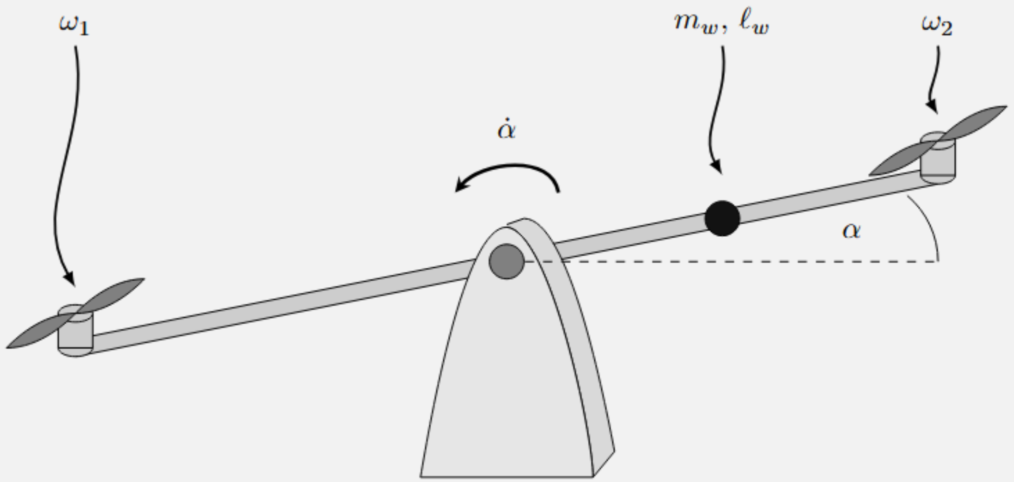
C) MISO

D) SIMO

HS 2017

Box 2: Questions 4, 5

You are enthusiastic about control systems and decide to learn more about it by building a seesaw as shown in the figure below. You connect two motors with propellers on either side of the seesaw to control the angle α of the seesaw. The propellers have angular velocities ω_1 and ω_2 as shown in the figure and have a distance ℓ from the center of the seesaw. An additional point mass with weight m_w and distance to the center of the seesaw ℓ_w is present.



A) Input: $\dot{\alpha}$, Output: ω_1, ω_2

B) Input: ω_1, ω_2, Output: α,

C) Input: α, Output: ω_1, ω_2

D) Input: $\dot{\alpha}$, Output: α

We can only influence the 2 motors
➞ Input
We want to control the angle
➞ Output

Theory Recap

Modeling

Remember our goals?

1. **Modeling:**
   – Learn how to represent a dynamic system in such a way that it can be treated effectively using mathematical tools.

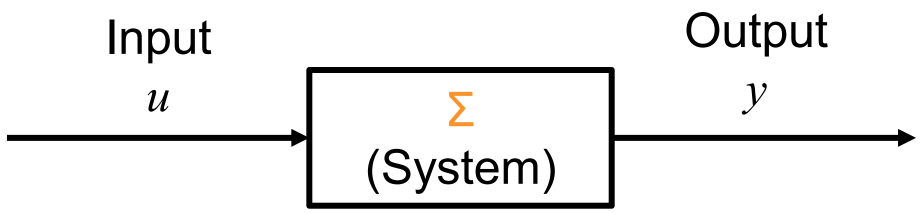
Today

2. **Analysis:**

3. **Synthesis:**

Modeling Steps

1. Identify the system boundaries
2. Write down the governing differential equations of the system.
3. Transform the system into the state-space form



1. System Boundaries

- How does outside world influence our system? These are Inputs!
  - Endogenous: can be manipulated by the designer, e.g. control inputs
  - Exogenous: generated by the environment and can't be controlled, e.g. disturbances.

- What do we observe about the system over time? These are Outputs!
  - Measured Outputs: what we can measure (sensors), e.g. speed of a car
  - Performance Outputs: not directly measurable, but we want to control, e.g. avg fuel consumption
- How does the system change internally over time? This is the state.
  - Also known as memory. It summarizes all the effects of past inputs.
- What are known quantities specific and constant to the system? These are parameters.

2. Differential Equations

- This step is very system dependet. How the ODE’s look like (Ordinary Differential Equations) needs to be determined by modeling the dynamics of the system. Often times however, this can be done by using a general storage approach.

$$\frac{d}{dt}\text{storage} = \Sigma\text{inflows} - \Sigma\text{outflows}$$

- Remember, our goal is to generally describe the how the system changes as a function of the current state.

$$\text{How the system changes} = \text{f(current state)}$$

3. State Space form

- The state space is just a convention we use to describe the system. It describes the changing system as well as the output as a function of other variables.

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= h(x(t), u(t)) \end{aligned}$$

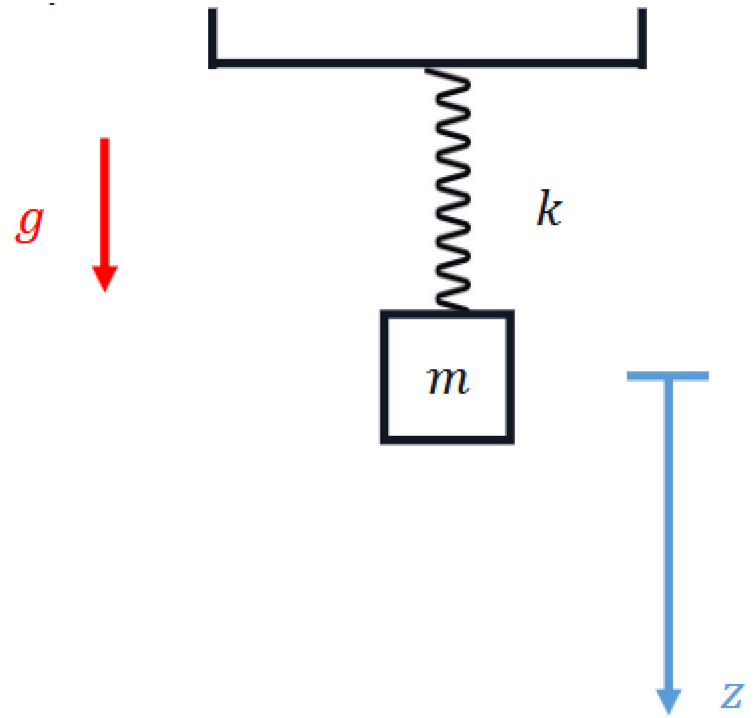
- Keep in mind that these are often matrices and vectors, meaning you might have a system of ODE’s

Spring Oscillator

Input: None
Output: Position
State: Position

Derive differential euqations
(Preview Dynamics)

$$F = m \cdot a$$
$$m\ddot{z} = mg - kz$$



Represent in State Space form:

Use: $x(t) = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

States: $\dot{x}(t) = \begin{pmatrix} \dot{z} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} x_2 \\ g - \frac{k}{m} x_1 \end{pmatrix}$

Output: $y(t) = z = x_1$

Problem 1

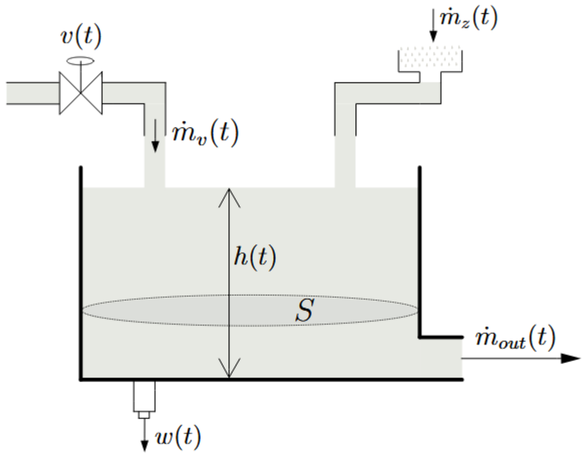


Figure 1: Water tank.

a) Identify the exogenous and endogenous inputs, the output y(t) and the state x(t). What are the disturbances d(t) and the control input u(t)? What could be an example for a parameter?

The tank has two inflows. The mass flow $\dot{m}_v(t)$ is regulated through a valve $v(t) \in [0, 1]$, such that $\dot{m}_v(t) = v(t)\dot{m}_{v,\max}$, where $\dot{m}_{v,\max}$ is a constant. The mass flow $\dot{m}_z(t)$ is generated by a rainfall collection system.
Assume that the outflow $\dot{m}_{out}(t)$ is a function of the tank geometry and the water height, $h(t)$ — i.e., we do not control the mass flow out, such as through a valve.
There is a sensor below the tank, whose signal, $w(t)$, measures the water level $h(t)$. The dynamics of the sensor are negligible, so we can assume $w(t) = h(t)$.
Finally, assume that the flows are frictionless and that water is an incompressible fluid of density ρ.

a) Identify the exogenous and endogenous inputs, the output y(t) and the state x(t). What are the disturbances d(t) and the control input u(t)

Endo: $v(t)$

Exo: $\dot{m}_{out}(t), \dot{m}_z(t)$

State: $x(t) = h(t)$

Control input: $u(t) = v(t)$

Disturbances $d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} \dot{m}_z(t) \\ \dot{m}_{out}(t) \end{bmatrix}$

Parameter: S

Problem 1

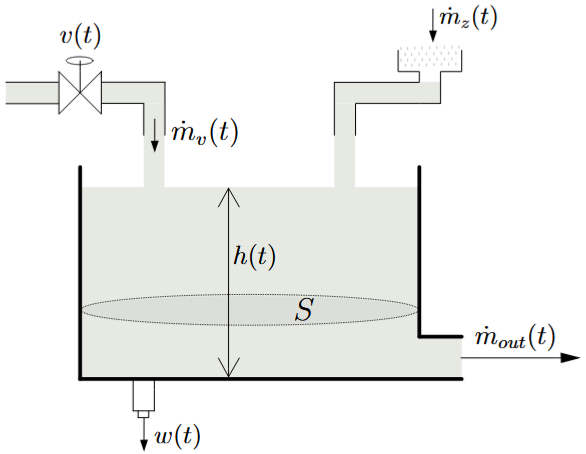


Figure 1: Water tank.

a) Identify the exogenous and endogenous inputs, the output y(t) and the state x(t). What are the disturbances d(t) and the control input u(t)?
b) What is the differential equation describing the evolution of the water height h(t)?

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b) Relate mass to water height:

$$V_{water} = S\,h(t),$$

$$V_{water}\,\rho = S\,h(t)\,\rho = m(t).$$

Because we are interested in the differential equation, we can determine the massflow

$$\begin{aligned}
\dot{m}(t) &= \sum \text{inflows} - \sum \text{outflows}, \\
\dot{m}(t) &= \dot{m}_v(t) + \dot{m}_z(t) - \dot{m}_{out}(t), \\
\dot{m}(t) &= v(t)\,\dot{m}_{v,\max} + \dot{m}_z - \dot{m}_{out}(t).
\end{aligned}$$

Now differentiating the second equation and plugging in the mass flow:

$$\dot{h}(t) = \frac{1}{S\,\rho}\left(v(t)\,\dot{m}_{v,\max} + \dot{m}_z(t) - \dot{m}_{out}(t)\right).$$

Problem 1

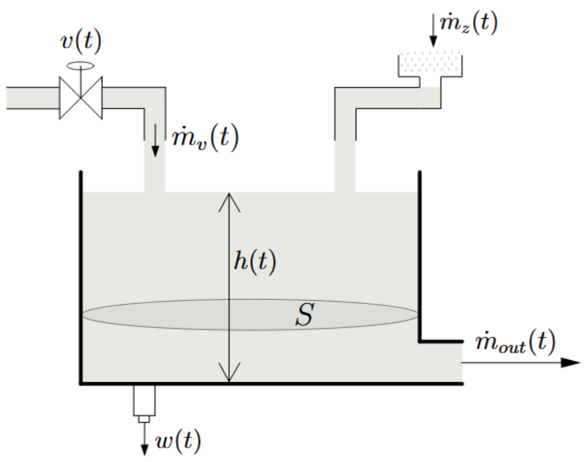

Figure 1: Water tank.

a) Identify the exogenous and endogenous inputs, the output y(t) and the state x(t). What are the disturbances d(t) and the control input u(t)?

b) What is the differential equation describing the evolution of the water height h(t)?

c) Write the system in state-space form

$$\dot{x}(t) = f\left(x(t), u(t), d(t)\right),$$
$$y(t) = g\left(x(t), u(t), d(t)\right).$$

The tank has two inflows. The mass flow $\dot{m}_v(t)$ is regulated through a valve $v(t) \in [0, 1]$, such that $\dot{m}_v(t) = v(t)\dot{m}_{v,\max}$, where $\dot{m}_{v,\max}$ is a constant. The mass flow $\dot{m}_z(t)$ is generated by a rainfall collection system.
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c) Write the system in state-space form

$$\dot{x}(t) = \frac{1}{S\,\rho}\left(u(t)\,\dot{m}_{v,\max} + d_1(t) - d_2(t)\right).$$

$$y(t) = x(t).$$

Block Diagrams

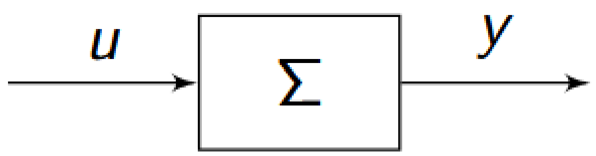
Block Diagrams

It is possible to have multiple systems, describeb by multiple states.

Block Diagrams help us to visually show how they are connected.

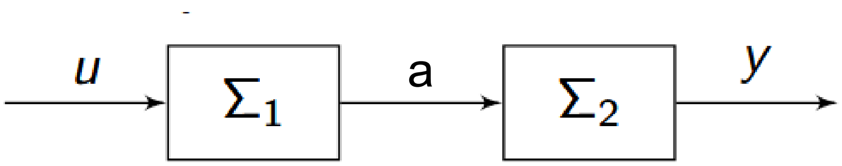
Often times we want to find the «overall System» or how it’s later called: «Transfer function». You’ll see what I mean… Let’s do some examples!

Basic



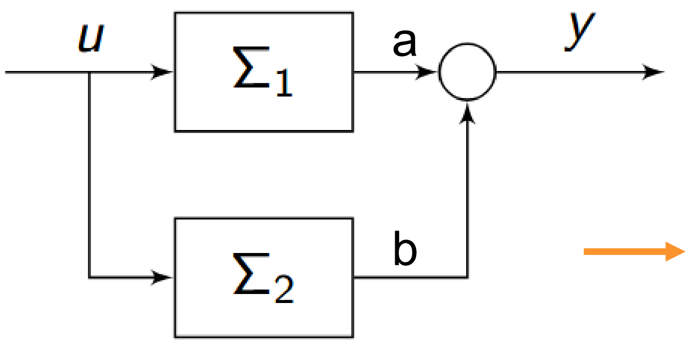
$$y = \Sigma u$$

Serie



$$\left.\begin{aligned} y &= \Sigma_2 a \\ a &= \Sigma_1 u \end{aligned}\right\} \quad y = \Sigma_2 \Sigma_1 u \qquad \Sigma = \Sigma_2 \Sigma_1$$

Parallel



$y = a + b$

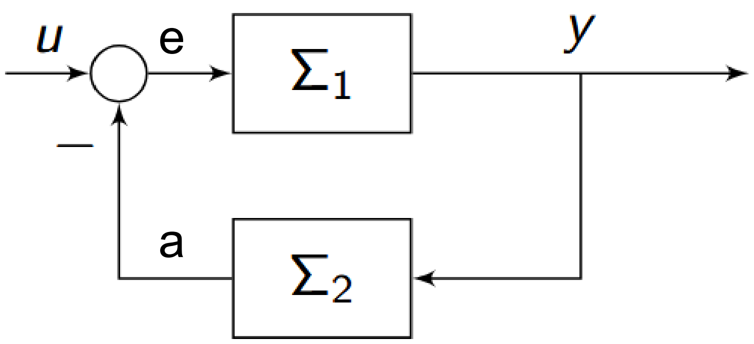
$a = \Sigma_1 u$

$b = \Sigma_2 u$

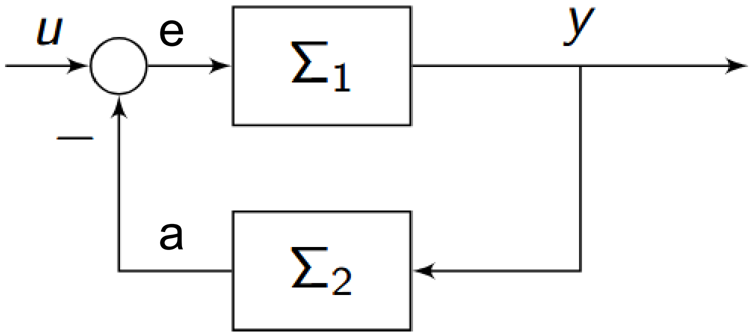
$\rightarrow \quad y = (\Sigma_1 + \Sigma_2)u$

$\rightarrow \quad \Sigma = \Sigma_1 + \Sigma_2$

Negative Feedback



Negative Feedback



$$y = \Sigma_1 e$$
$$e = u - a$$
$$a = \Sigma_2 y$$

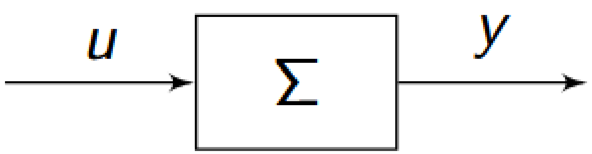
$$y = \Sigma_1(u - \Sigma_2 y)$$
$$y = \Sigma_1 u - \Sigma_1 \Sigma_2 y$$
$$y + \Sigma_1 \Sigma_2 y = \Sigma_1 u$$
$$(1 + \Sigma_1 \Sigma_2) y = \Sigma_1 u$$

$$\Rightarrow y = (1 + \Sigma_1 \Sigma_2)^{-1} \cdot \Sigma_1 u$$

$$\Sigma = (1 + \Sigma_1 \Sigma_2)^{-1} \Sigma_1$$

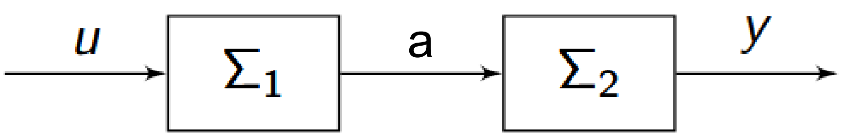
$$\rightarrow \Sigma = \frac{\Sigma_1}{(1+\Sigma_1\Sigma_2)}$$

Basic



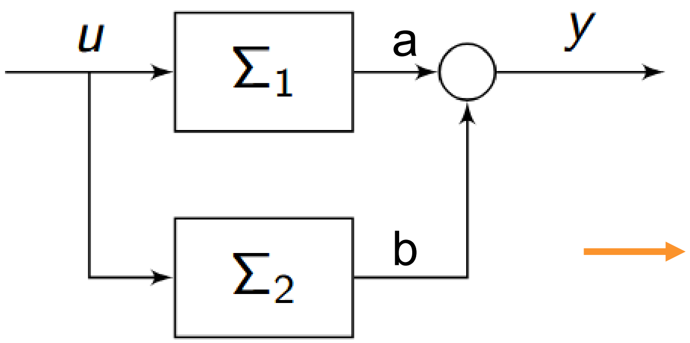
$$y = \Sigma u$$

Serie



$$\left.\begin{array}{l} y = \Sigma_2 a \\ a = \Sigma_1 u \end{array}\right\} \quad y = \Sigma_2 \Sigma_1 u \qquad \Sigma = \Sigma_2 \Sigma_1$$

Parallel



$y = a + b$

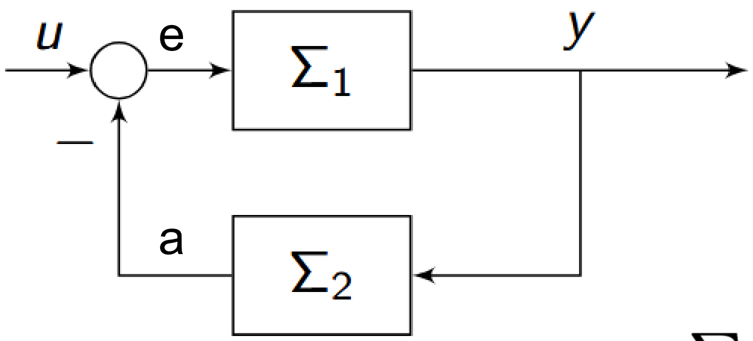
$a = \Sigma_1 u$

$b = \Sigma_2 u$

→ $y = (\Sigma_1 + \Sigma_2) u$

→ $\Sigma = \Sigma_1 + \Sigma_2$

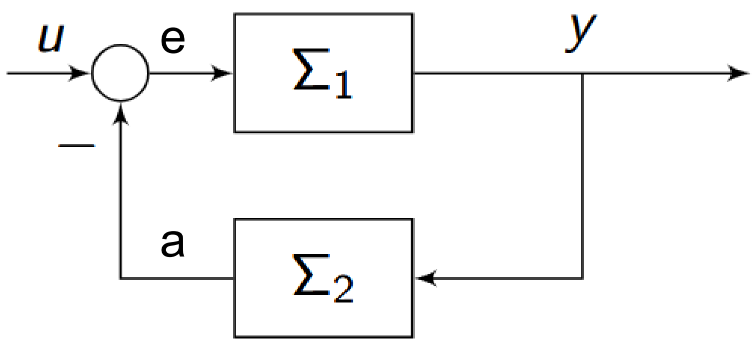
Negative Feedback



→ $\Sigma = \frac{\Sigma_1}{(1 + \Sigma_1 \Sigma_2)}$

Unstable?

Let's have look at the feedback case again. We heard that a feedback architecture can become «unstable». But what does that really mean?



$$\Sigma = \frac{\Sigma_1}{(1+\Sigma_1\Sigma_2)}$$

Remember that the overall system can be used for the output:

$$y = \Sigma u = \frac{\Sigma_1}{(1+\Sigma_1\Sigma_2)} u$$

But what happens now as $\Sigma_1\Sigma_2 \to -1$?

$$y \to \infty$$

The output explodes and becomes unstable!!

HS 2022

Problem: Consider the interconnected system shown in Figure 4. You can assume that the input-output relation for each system Σ_i is given by $y_i = \Sigma_i \cdot u_i$, that is, each of the systems Σ_i represents a simple scalar gain.

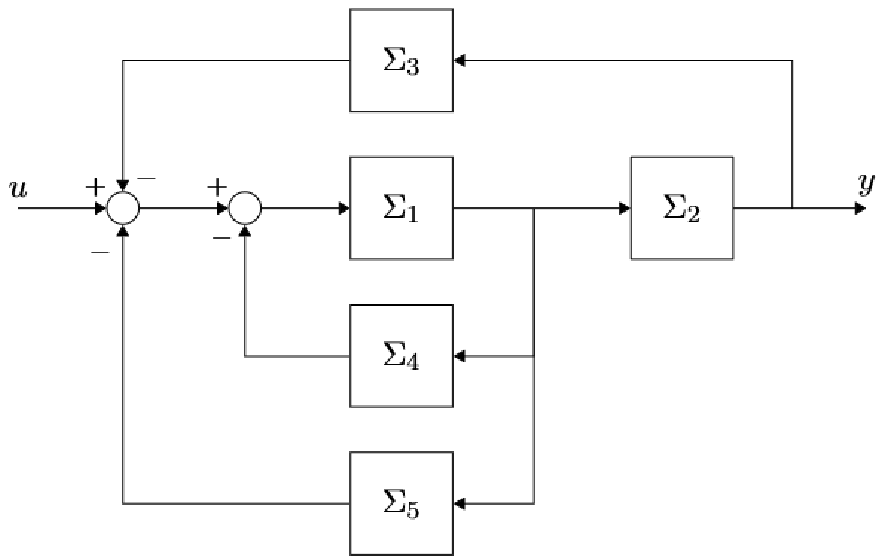


Figure 4: Interconnected system.

Q4 **(1.5 Points)** Derive the transfer function Σ from u to y for the interconnected system shown in Figure 4. **Simplify the result as much as possible.**

$\Sigma =$ ____________________________

$$y = \Sigma_2 a, \qquad a = \Sigma_1 b, \quad b = d - c, \quad c = \Sigma_4 a,$$
$$d = u - e - f, \quad e = \Sigma_3 y, \quad f = \Sigma_5 a.$$

$$\begin{aligned} d &= u - \Sigma_3 y - \Sigma_5 a, \\ b &= d - c = u - \Sigma_3 y - \Sigma_5 a - \Sigma_4 a, \\ a &= \Sigma_1 b = \Sigma_1\big(u - \Sigma_3 y - \Sigma_5 a - \Sigma_4 a\big). \end{aligned}$$

$$\begin{aligned} a + \Sigma_1\Sigma_5 a + \Sigma_1\Sigma_4 a &= \Sigma_1(u - \Sigma_3 y), \\ (1 + \Sigma_1\Sigma_5 + \Sigma_1\Sigma_4)\, a &= \Sigma_1(u - \Sigma_3 y), \\ a &= \frac{\Sigma_1(u - \Sigma_3 y)}{1 + \Sigma_1\Sigma_5 + \Sigma_1\Sigma_4}. \end{aligned}$$

$$\begin{aligned} y = \Sigma_2 a &= \frac{\Sigma_2\Sigma_1 u - \Sigma_2\Sigma_1\Sigma_3 y}{1 + \Sigma_1\Sigma_5 + \Sigma_1\Sigma_4}, \\ \left(1 + \frac{\Sigma_2\Sigma_1\Sigma_3}{1 + \Sigma_1\Sigma_5 + \Sigma_1\Sigma_4}\right) y &= \frac{\Sigma_2\Sigma_1}{1 + \Sigma_1\Sigma_5 + \Sigma_1\Sigma_4}\, u, \\ y &= \frac{\Sigma_2\Sigma_1}{1 + \Sigma_1\Sigma_5 + \Sigma_1\Sigma_4 + \Sigma_2\Sigma_1\Sigma_3}\, u. \end{aligned}$$

Feedback

https://n.ethz.ch/~jschul/Feedback

Q&A Session / Done